Name and Surname : Solns

Grade/Class : 12/……… **Mathematics Teacher** : ……………………

____
150

NB
Euclidean Geo reasons
• must be correctly stated
• only earn a mark if correctly applied in the diagram

PAPER 2
June 2016
ANSWER BOOKLET

1.

Standardised Test 1 result % (x)	Volume of the balloon cm^3 (y)
55	4 100
40	4 800
75	5 000
80	6 000
65	3 800
90	4 000

1.1.1.	$A = 4214,76$ ✓	
	$B = 5,95$ ✓	
	$\therefore\ y = 4214,76 + 5,95x$ ✓	3
1.1.2.	$r = 0,13$ ✓	1
1.2.	As x increases, y increases ✓	1
	(or: as test result increases, the	
	volume increases)	

1.3.1.	$y = 4214,76 + 5,95(85)$	
	$= \underline{4\,720,51 \text{ cm}^3}$ ✓ no penalty if no units	1
1.3.2.	$r \approx 0,1$	
	$\therefore$ the correlation between the	
	variables is very weak ✓	
	$\therefore$ the prediction is not reliable. ✓	2

2.

2.1.	$\bar{x} = \frac{700}{35}$	
	$= \underline{20}$ ✓	1
2.2.	variance	
	$= \sigma^2$	
	$= \frac{140}{35}$	
	$= \underline{4}$ ✓ if $\sqrt{4} = 2$ 0/1	1

3.

Data value x	Frequency f
0	2
1	15
2	22
3	37 — 76
4	30 — 106
5	20
6	13
7	10
8	8
9	5
10	3

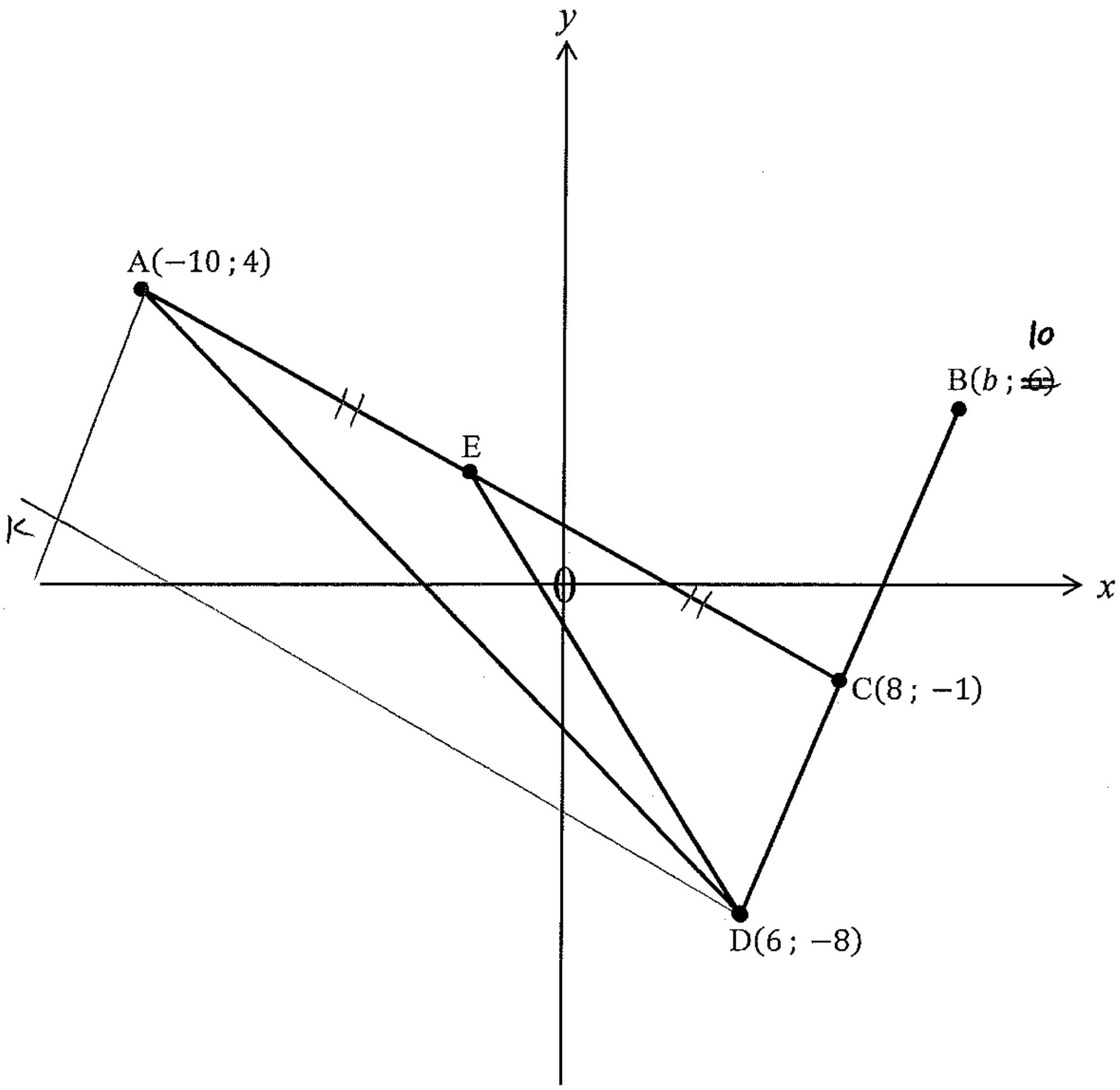
frequency column on/off

- (shift) SETUP
- ↓
- STAT
- ON

3.1.1.	$n = 165$ ✓	1
3.1.2.	$\bar{x} = 4,1$ ✓	1
3.1.3.	$\sigma = 2,22$ ✓	1
3.1.4.	$\bar{x} \pm 0,45\sigma$	
	$4,1 - 0,45 \cdot 2,22$ $= 3,101$; $4,1 + 0,45 \cdot 2,22$ $= 5,099$	
	$\therefore$ 4 : 30 ✓	
	5 : 20	
	$\therefore$ 50 values ✓ answ only 1/2	2
	• CA using 3.1.2 ($\bar{x}$) and 3.1.3. (σ)	
	• if no 3.1.2. or 3.1.3. 0/2	

3.1.5.	1. $M = T_{\frac{1}{2}(1+165)}$	
	$= T_{83}$ ✓ ie 83	1
3.1.5.	2. $M = 4$ ✓	1
3.1.6.	$D_4 = T_{\frac{4}{10}(1+165)}$	
	$= T_{66,4}$ ✓ ie 66,4	1
3.1.7.	$M = T_{83}$ $\therefore$ $T_{84}; \dots ; T_{165}$	
	$\therefore Q_3 = T_{\frac{1}{2}(84+165)}$	
	$= T_{124,5}$ ✓ ie 124,5	1
3.2.	$\bar{x} - M = 4,1 - 4$ ✓	
	$= 0,1$	
	> 0	
	$\therefore$ the data is positively skewed ✓	
	(skewed to the right)	2
	• answ only ½	
	• ca using 3.1.2. ($\bar{x}$) and 3.1.5.2. (M)	
	• if no 3.1.2. or 3.1.5.2. 0/2	

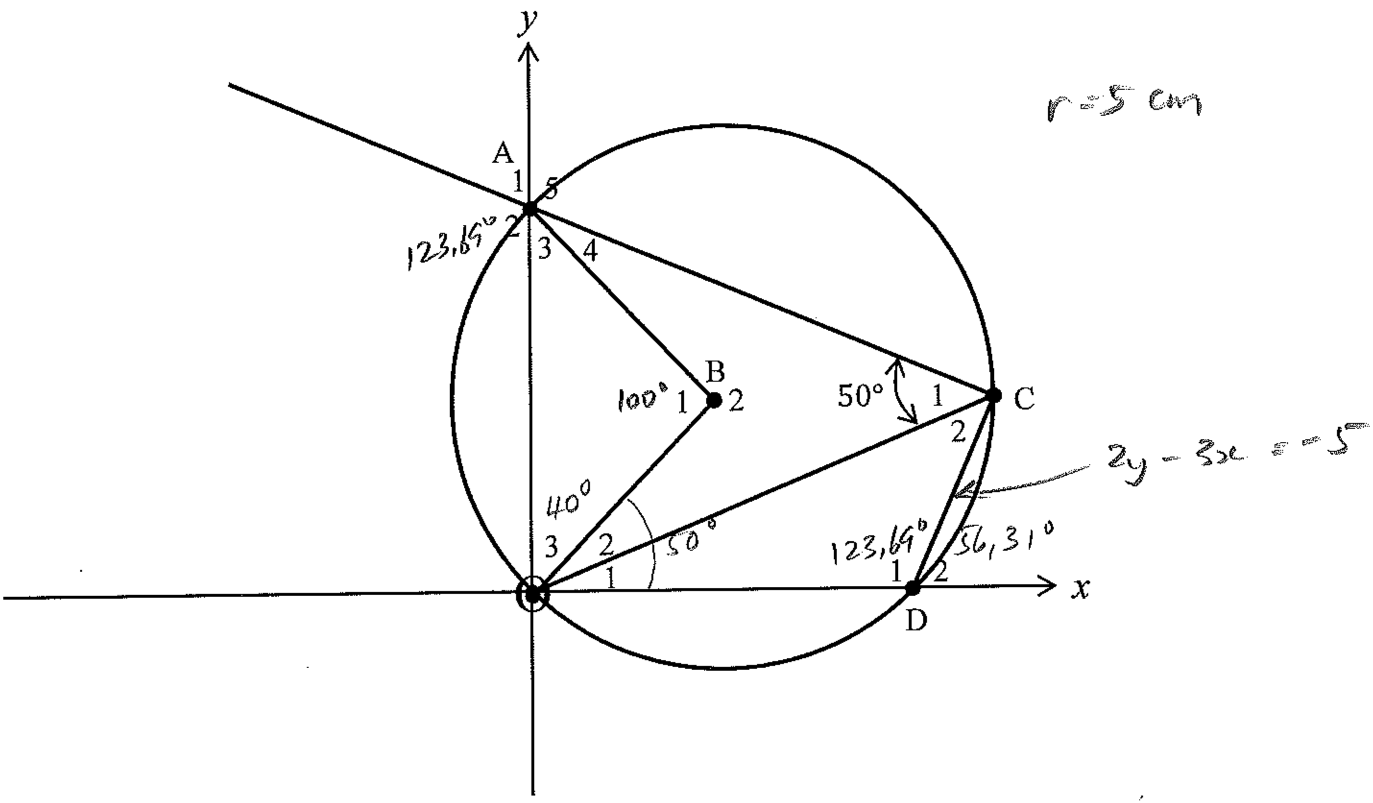
4.



4.1.	AE = EC DE is a median	
	A(−10;4) E C(8;−1)	
	$x_E = \frac{-10+(8)}{2}$ $y_E = \frac{4+(-1)}{2}$	
	$= -1$ $= 3/2$	
	$\therefore$ E(−1; 3/2)	2
	✓ ✓	
4.2.	D(6;−8) C(8;−1) B(b;10)	
	$m_{DC} = \frac{-1-(-8)}{8-6} = \frac{7}{2}$ ✓	
	$m_{CB} = \frac{10-(-1)}{b-8} = \frac{11}{b-8}$ ✓	

	$\frac{7}{2} = \frac{11}{b-8}$ ✓ method	
	$\therefore 7(b-8) = 11(2)$ $\quad \therefore$ $b = \frac{78}{7}$ ✓	4
4.3.	$C(8;-1) \quad F(f;-5) \quad D(6;-8)$	
	$CF = \sqrt{(-5-(-1))^2 + (f-8)^2} = \sqrt{16 + (f-8)^2}$ ✓	
	$CD = \sqrt{(-8-(-1))^2 + (6-8)^2} = \sqrt{53}$ ✓	
	$CF = CD \quad \therefore \quad \sqrt{16+(f-8)^2} = \sqrt{53}$	
	$()^2$ bs $\quad 16 + (f-8)^2 = 53$ ✓	
	$(f-8)^2 = 37$	
	$f - 8 = \pm\sqrt{37}$ ✓	
	$f = 8 \pm \sqrt{37}$	
	$= 14{,}08$ or $1{,}92$ ✓ ✓	6
4.4.	$C(8;-1) \xrightarrow[5\uparrow]{18\leftarrow} A(-10;4)$	
	$D(6;-8) \xrightarrow[5\uparrow]{18\leftarrow} K(-12;-3)$ ✓ ✓	2

5.



5.1.	$\hat{B}_1 = 100°$ ✓ ✓ ∠ @ centre = 2∠ @ O'ce	
	∴ area Δ ABC	
	$= \frac{1}{2}(5)(5)\sin 100°$ ✓ f+s	
	$= 12,31 \text{ cm}^2$ ✓	4
5.2.	$2y - 3x = -5 \quad \therefore \quad y = \frac{3}{2}x - \frac{5}{2}$	
	$\therefore \quad m_{CD} = 3/2$	
	$\therefore \quad \tan \hat{D}_2 = 3/2$ ✓	
	ref ∠ $= 56,30\ldots°$	
	tan + in I	
	I : $\hat{D}_2 = 56,31°$ ✓	2

5.3.	$\hat{D}_1 = 123,69°$ ✓SR ∠'s str line $= 180°$	
	$\therefore \hat{A}_2 = 123,69°$ ext ∠ cyclic quad.	3
	✓S ✓R	
5.4.	$\hat{O}_3 = \hat{A}_2$ ✓SR ∠'s opp $=$ sides, radii	
	$\therefore \hat{O}_3 = 40°$ ✓SR ∠'s $\Delta = 180°$	
	$\therefore \hat{O}_{1+2} = 50°$ ✓S	3
5.5.	$\cos 50° = \frac{x_B}{5}$	
	$5 . \cos 50° = x_B$	
	$3,21 =$ ✓	1

6.

6.1. $\cos 18^\circ = m$

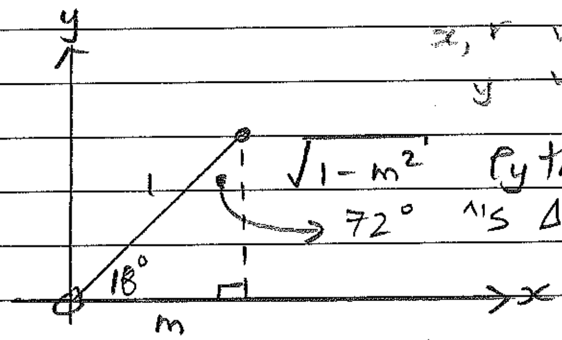
6.1.1. $\cos(-18^\circ) = \cos 18^\circ$

$= m$ — 1

6.1.2. $\cos 18^\circ = \frac{m}{1}$ $\quad \frac{x}{r}$ $\quad$ QI

x, r

y



Pyth

$\angle$'s $\Delta = 180^\circ$

$\tan 108^\circ$

$= \tan(180^\circ - 72^\circ)$

$= -\tan 72^\circ$

$= -\frac{m}{\sqrt{1-m^2}}$ $\quad \frac{o}{a}$

OR

$\tan 108^\circ$

$= \tan(90^\circ + 18^\circ)$

$= \cancel{\frac{\sin(90^\circ + 18^\circ)}{\cos(90^\circ + 18^\circ)}}$

$= \frac{\cos 18^\circ}{-\sin 18^\circ}$

$= \frac{m}{\frac{-\sqrt{1-m^2}}{1}}$

$= -\frac{m}{\sqrt{1-m^2}}$ — 4

6.1.3. $\cos 2208^\circ$

$= \cos 48^\circ$

$= \cos(30^\circ + 18^\circ)$

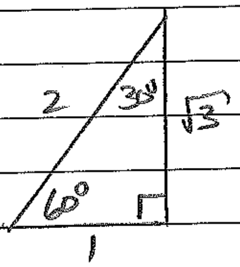
$= \cos 30^\circ \cos 18^\circ - \sin 30^\circ \sin 18^\circ$

NB
30°

$= \left(\frac{\sqrt{3}}{2}\right)(m) - \left(\frac{1}{2}\right)\left(\sqrt{1-m^2}\right)$ ✓ 18°

$= \frac{\sqrt{3}m - \sqrt{1-m^2}}{2}$

NB



4

6.1.4

$\cos 2x = 1 - 2\sin^2 x$

$\cos 18^\circ = 1 - 2\sin^2 9^\circ$ ✓

$m = 1 - 2\sin^2 9^\circ$

$\therefore \sin 9^\circ = \sqrt{\frac{m-1}{-2}}$ ✓ $\pm$ reject

2

6.2.1.

$\text{LHS} = \frac{\sin 2x + 1}{\cos 2x}$

$= \frac{2\sin x\cos x + \sin^2 x + \cos^2 x}{\cos^2 x - \sin^2 x}$ ✓ ✓ ✓

$= \frac{\cos^2 x + 2\sin x\cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \frac{(\cos x + \sin x)(\cos x + \sin x)}{(\cos x + \sin x)(\cos x - \sin x)}$ ✓ ✓

$= \frac{\cos x + \sin x}{\cos x - \sin x}$

$= \text{RHS}$

5

6.2.2.

$\cos 2x = 0$ and $\cos x - \sin x = 0$

$A = 2x$ $\quad 1 - \tan x = 0$

$\cos A = 0$ $\quad \tan x = 1$

$A = 90^\circ + k180^\circ$ $\quad$ ref $\hat{}$ $= 45^\circ$

$2x =$ $\quad$ tan + in I

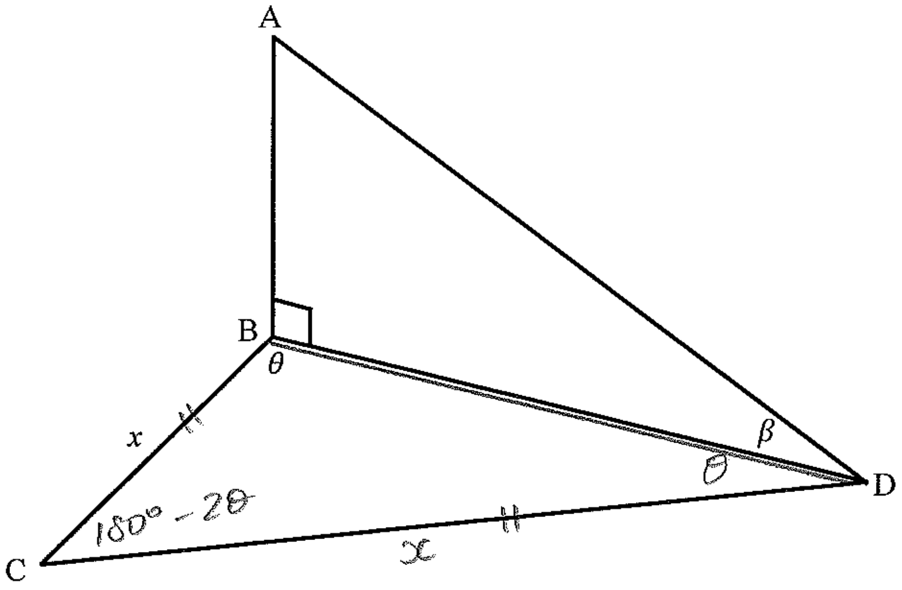
$x = 45^\circ + k90^\circ$ ✓ ✓ $\quad$ I: $x = 45^\circ + k180^\circ$

$(k \in \mathbb{Z})$

2

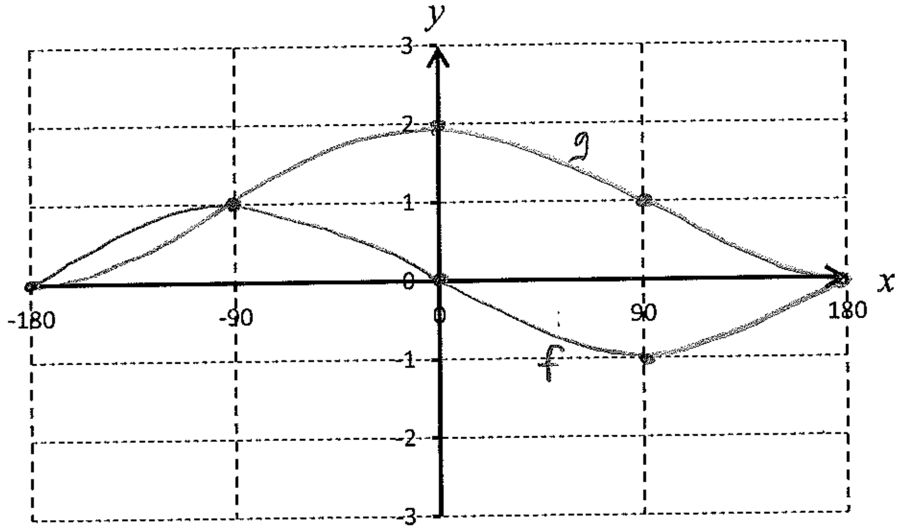
6.3.	$\dfrac{\cos 170^\circ \cos 30^\circ + \cos 280^\circ \sin 30^\circ}{\sin 25^\circ \cos 25^\circ}$	
	170° Q II $180^\circ - 10^\circ$, $90^\circ + 80^\circ$ 280° Q IV $360^\circ - 80^\circ$, $270^\circ + 10^\circ$	
	• $\cos 170^\circ = \cos(90^\circ + 80^\circ)$	
	$= -\sin 80^\circ$ ✓	
	• $\cos 280^\circ = \cos(360^\circ - 80^\circ)$	
	$= \cos 80^\circ$ ✓	
	∴ num	
	$= -\sin 80^\circ . \cos 30^\circ + \cos 80^\circ \sin 30^\circ$	
	$= -(\sin 80^\circ \cos 30^\circ - \cos 80^\circ \sin 30^\circ)$ ✓	
	$= - \sin(80^\circ - 30^\circ)$ $\sin(-50^\circ)$	
	$= -\sin 50^\circ$ ✓	
	$= -2\sin 25^\circ \cos 25^\circ$ ✓	
	$\therefore \dfrac{-2\sin 25^\circ \cos 25^\circ}{\sin 25^\circ \cos 25^\circ} = -2$ ✓	6
6.4.	$3\cos 2x = 1 + 5\cos x$	
	✓ $3(2\cos^2 x - 1) = 1 + 5\cos x$	
	$6\cos^2 x - 3 = 1 + 5\cos x$	
	$6\cos^2 x - 5\cos x - 4 = 0$ ✓	
	$(3\cos x - 4)(2\cos x + 1) = 0$ ✓	
	$\therefore \cos x = \frac{4}{3}$ or ✓ $\cos x = -\frac{1}{2}$	
	no sol ✓ ref ∠ $= 60^\circ$	
	$\cos$ − in	
	$(k \in \mathbb{Z})$ ✓ II: $x = 120^\circ + k\,360^\circ$ ✓	
	III: $x = 240^\circ + k\,360^\circ$ ✓	8

7.



7.1.	$C\hat{D}B = \theta$ ∠'s opp = sides ✓SR	
	$\hat{C} = 180° - 2\theta$ ∠'s Δ = 180° ✓SR	
	$\frac{BD}{\sin(180° - 2\theta)} \overset{✓}{=} \frac{x}{\sin\theta}$	
	$\frac{BD}{✓\sin 2\theta} = \frac{x}{\sin\theta}$	
	$BD = \frac{x \cdot \sin 2\theta}{\sin\theta}$	
	$= \frac{x \cdot 2\sin\theta\cos\theta}{\sin\theta}$ ✓	
	$= 2x\cos\theta$	5
7.2.	$\frac{AB}{BD} = \tan\beta$ ✓	
	$\therefore AB = BD \cdot \tan\beta$	
	$= 2x\cos\theta\tan\beta$ ✓	2

8. $f(x) = -\sin x$ and $g(x) = \cos x + 1$

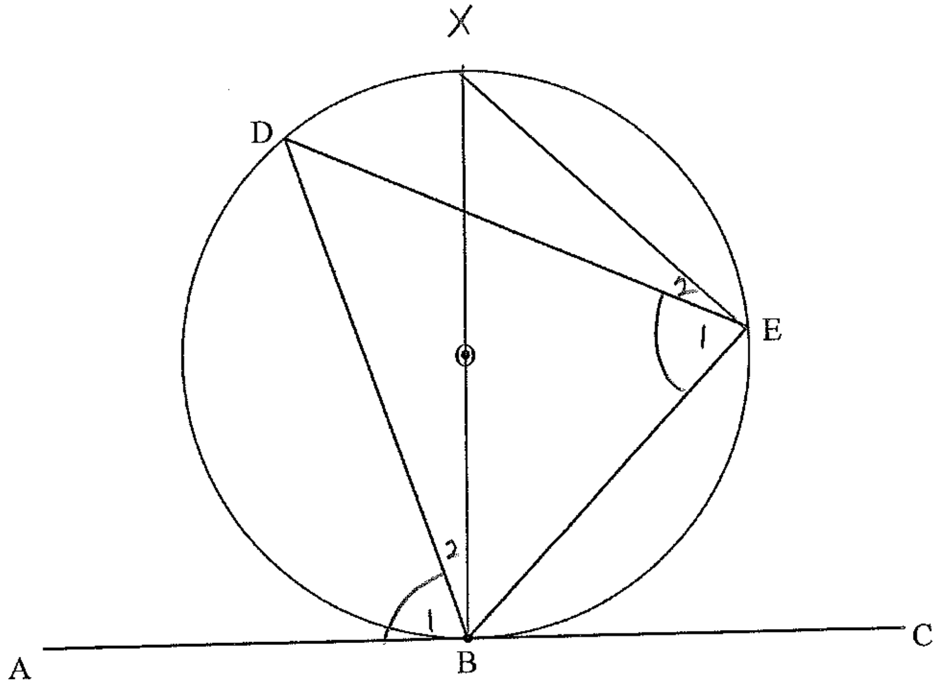


	f		g
x ints	✓	✓	x int
max/min	✓	✓	max
shape	✓	✓	shape

8.1.	f: $y = -\sin x$ g: $y = \cos x + 1$	6
8.2. 1.	f decreasing down L → R	
	$x \in (-90°; 90°)$ ✓	1
8.2. 2.	$\cos x + 1 + \sin x = 2$	
	$\cos x + 1 - (-\sin x) = 2$	
	$y_g - y_f = 2$	
	$x = 0°$ or $90°$	2
	✓ ✓	
8.2. 3.	$(-\sin x)(\cos x + 1) \geq 0$	
	$y_f \cdot y_g$ $\overset{+}{0}$	
	$\therefore x \in [-180°; 0°]$ or $x = 180°$	2
	✓ ✓	

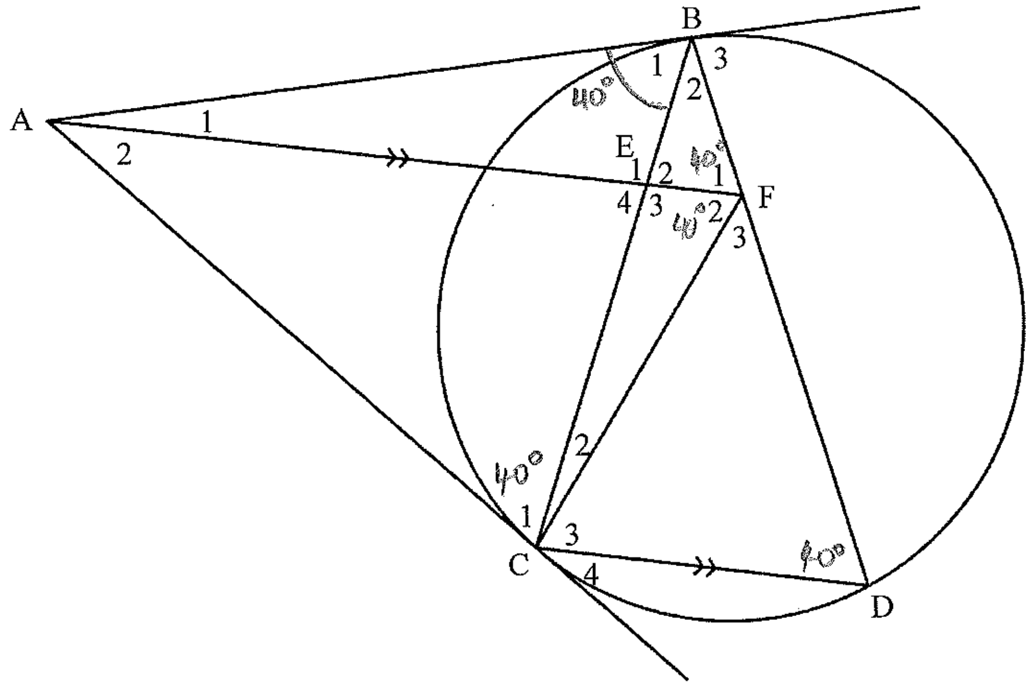
8.2.	4. $\cos x + 1 + \sin x > 0$	
	$\cos x + 1 > -\sin x$	
	$y_g > y_f$	
	$\therefore x \in (-90°; 180°)$	1
8.3.	$2\sin^2 x - \sin x + 2\cos^2 x$	
	$= -\sin x + 2(\sin^2 x + \cos^2 x)$	
	$= -\sin x + 2.1$	
	$= -\sin x + 2$	
	$= 1 + 2$	
	$= 3$	2

9.1.



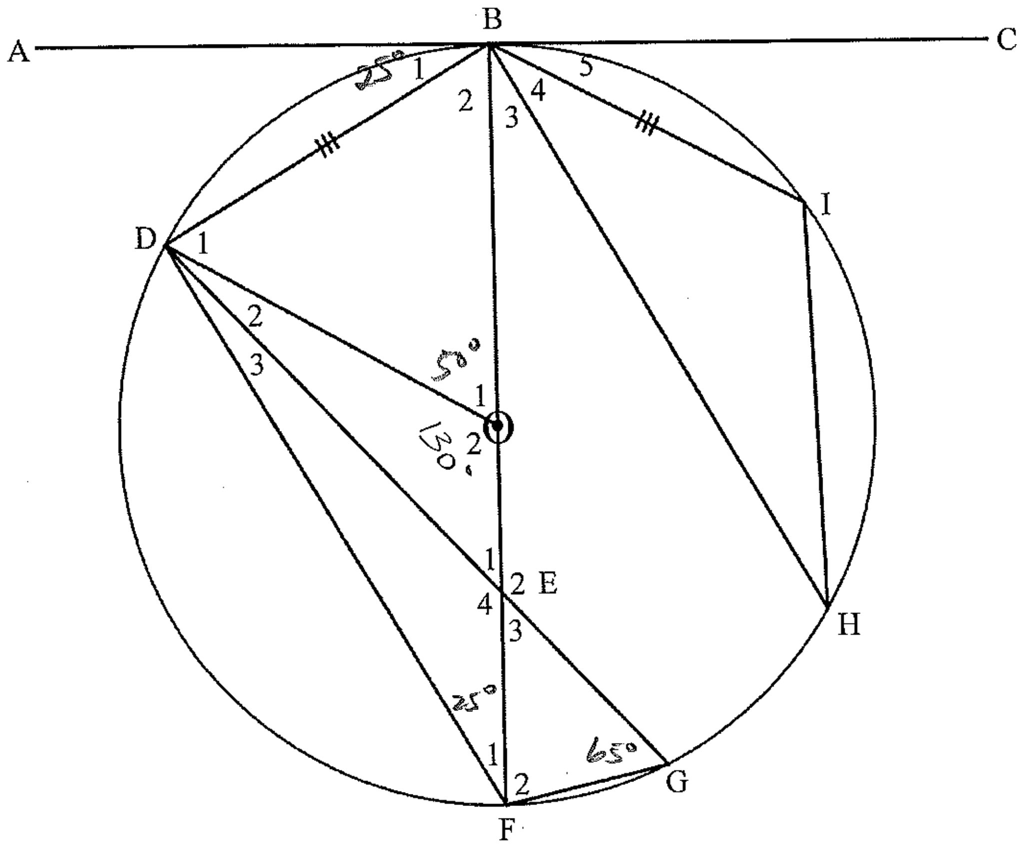
	Constr BOX and XE ✓	
	$\hat{E}_1 + \hat{E}_2 = 90^\circ$ ✓SR ∠ in semi ⊙ $= 90^\circ$	
	$\hat{B}_1 + \hat{B}_2 = 90^\circ$ ✓SR tan ⊥ rad	
	but $\hat{B}_2 = \hat{E}_2$ ✓SR ∠s in same ⊙ segm =	
	∴ $\hat{B}_1 = \hat{E}_1$	
	ie $A\hat{B}D = D\hat{E}B$	4

9.2.



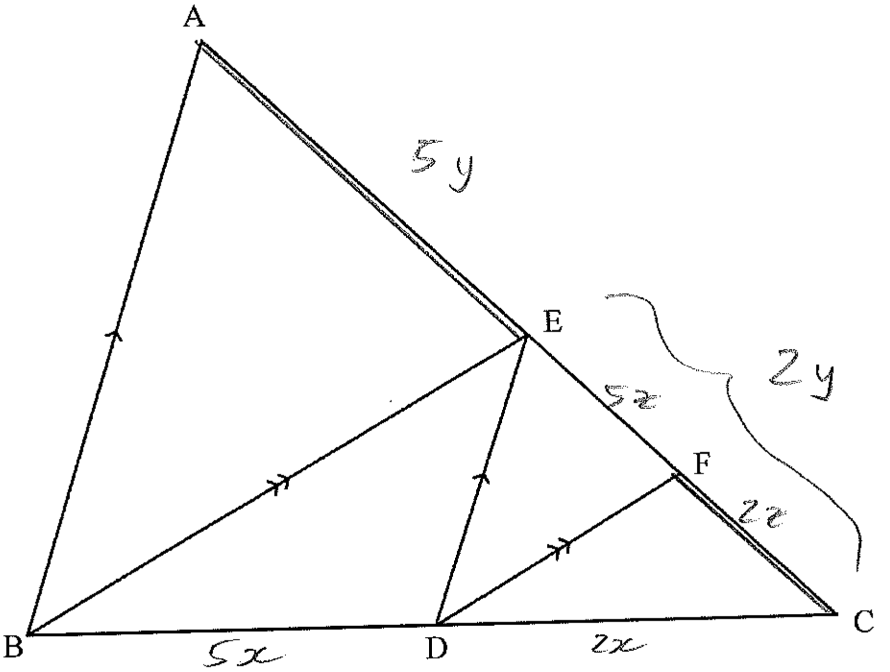
9.2.	1. $\hat{D} = 40°$ ✓S ✓R ∠ tan chord	
	∴ $\hat{F}_1 = 40°$ ✓SR corr ∠s =, AF ∥ CD	3
9.2.	2 $\hat{C}_1 = 40°$ ✓S ✓R ∠ tan chord	
	∴ $\hat{C}_1 = \hat{F}_1$ ✓S both = 40°	
	∴ ABFC is a cyclic quad conv ∠s in same ⊙ segm = ✓R	4
9.2.	3. $\hat{F}_2 = 40°$ ✓SR ∠'s in same ⊙ segm =	
	∴ $\hat{F}_1 = \hat{F}_2$ ✓S both = 40°	
	∴ AF bisects $B\hat{F}C$	2

10.



10.1.	$\hat{F}_1 = 25°$ ✓S ✓R ∠ tan chord	
	$\therefore \hat{O}_1 = 50°$ ✓S ✓R ∠ @ centre = 2∠ @ ⊙'ce	4
10.2.	$\hat{O}_2 = 130°$ ✓SR ∠'s str line = 180°	
	$\therefore \hat{G} = 65°$ ✓SR ∠ @ centre = 2∠ @ ⊙'ce	2
10.3.	$\hat{H} = 25°$ ✓S ✓R = chords, = ∠'s	2

11.



S P

$\frac{AE}{EC} = \frac{5}{2}$ line ∥ 1 side Δ

S

$\frac{CF}{FE} = \frac{2}{5}$ line ∥ 1 side Δ

$\therefore \frac{AE}{FC} = \frac{5y}{2z}$

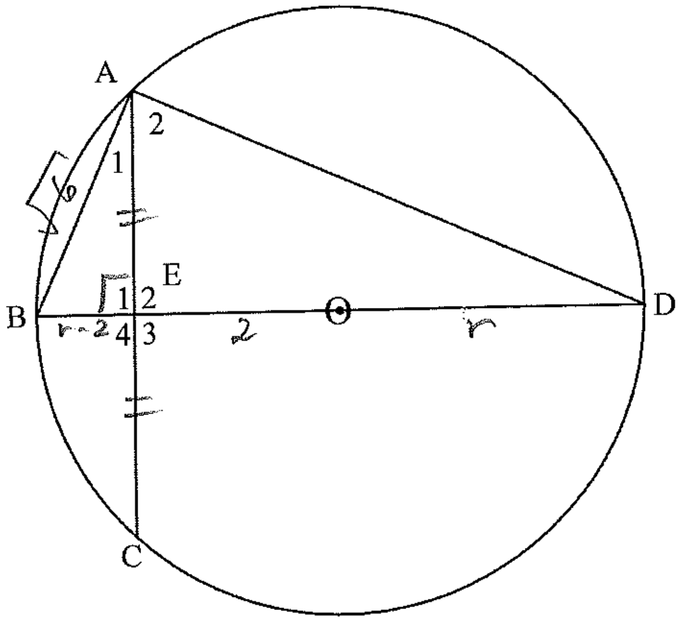
$= \frac{5(\frac{7}{2}z)}{2z}$ $2y = 7z$ $\therefore y = \frac{7}{2}z$

$= \frac{35}{4}$ → 8,75

4

12.1.

r



12.1.	1. In Δ's $A_1\,B\,E_1$, $D\,B\,A_{1+2}$	
	1. $\hat{B} = \hat{B}$ ✓SR common	
	2. $\hat{E}_1 = 90°$ ✓SR line centre ⊙ to mdpt chord is ⊥	
	$\hat{A}_{1+2} = 90°$ ✓SR ∠ in semi ⊙ = 90°	
	$\therefore\ \hat{E}_1 = \hat{A}_{1+2}$ ✓S both = 90°	
	$\therefore\ \Delta ABE \parallel\!\mid \Delta DBA$ AAA ✓	5
12.1.	2. BE = r − 2 ✓ OB = r	1
12.1.	3. $\frac{AB}{DB} = \frac{BE}{BA}$ ✓S ||| Δ's	
	$\therefore\ AB^2 = BE \cdot BD$	

$\therefore \quad (\sqrt{6})^2 = (r-2)(2r)$ ✓

$6 = 2r^2 - 4r$

$0 = 2r^2 - 4r - 6$

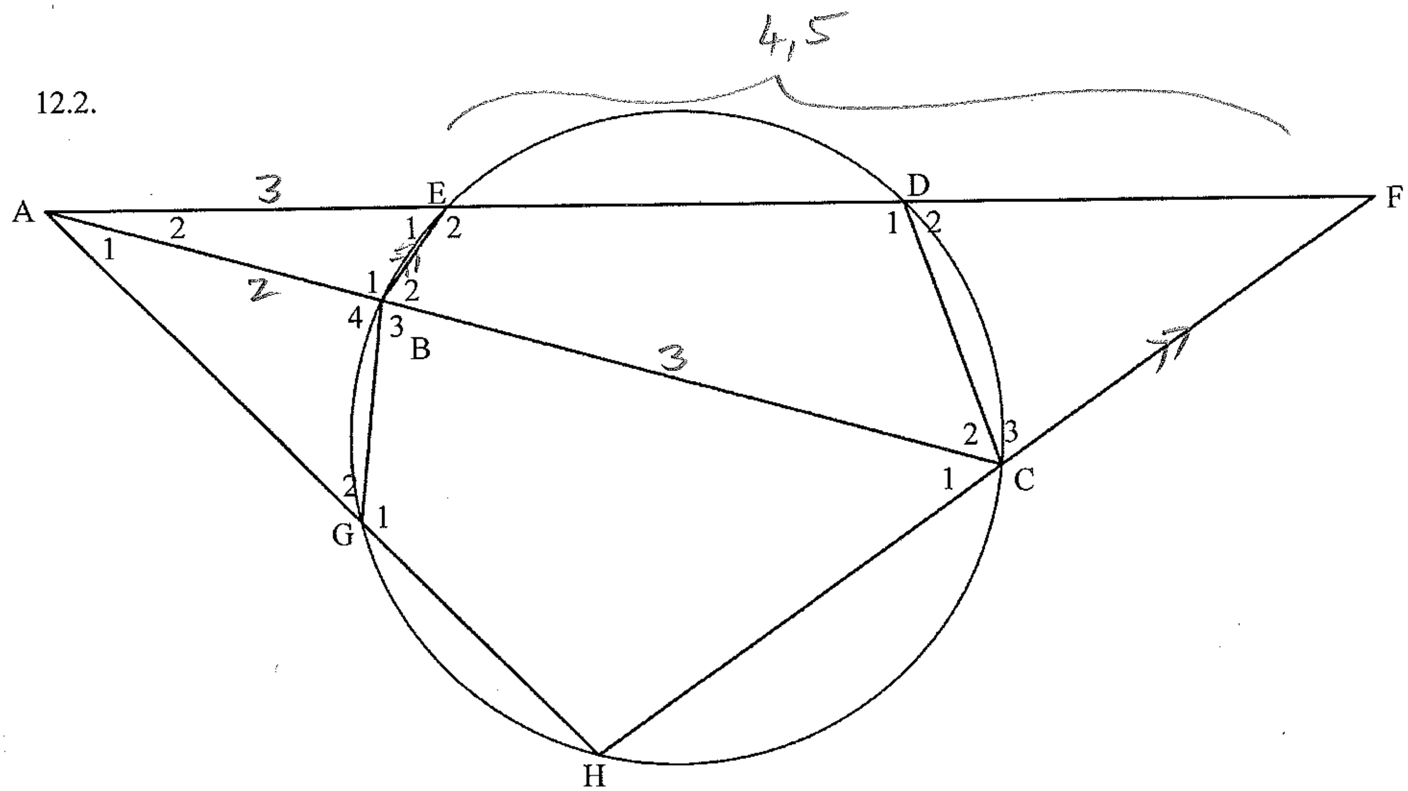
$\div 2: \quad 0 = r^2 - 2r - 3$ ✓

$0 = (r+1)(r-3)$ ✓

$\therefore \quad r = \cancel{-1}$ or 3 ✓ -1 must be rejected | 5

reject $\longrightarrow$

12.2.



12.2.	1. $\frac{AE}{EF} = \frac{3}{4,5} = \frac{2}{3}$ ✓S	
	$\frac{AB}{BC} = \frac{2}{3}$	
	$\therefore \frac{AE}{EF} = \frac{AB}{BC}$ ✓S both $= \frac{2}{3}$	
	$\therefore$ BE ∥ CF ✓R line ÷ 2 sides Δ in prop^n	3
12.2.	2. In Δ's $A_2 B_1 E_1$, $A_2 C_{2+3} F$	
	1. $\hat{A}_2 = \hat{A}_2$ ✓SR common	
	2. $B_1 = C_{2+3}$ ✓SR corr ∠'s =, BE ∥ CF	
	$\therefore$ Δ ABE ∥∥ Δ ACF ✓S AAA ✓R	
	$\therefore \frac{AB}{AC} = \frac{BE}{CF}$ ∥∥ Δ's	4

12.2.	3	<u>Planning</u> :	
		$\frac{AH.BG}{HC} = \underbrace{\frac{BE.AC}{CF}}_{AB}$ 12.2.2	
		$AH.BG = AB.HC$	
		$\frac{AH}{AB} = \frac{HC}{BG}$	
		$\therefore AHC, ABG$	
		<u>Answer</u>	
		In Δ's $A_1 H C_1$, $A_1 B_4 G_2$	
		1. $\hat{A}_1 = \hat{A}_1$ ✓S/R common	
		2. $\hat{H} = \hat{B}_4$ ✓S/R ext $\angle$ cyclic quad	
		$\therefore \Delta AHC \mathbin{\vert\vert\vert} \Delta ABG$ ✓S AAA ✓R	
		$\therefore \frac{AH}{AB} = \frac{HC}{BG}$ ✓S $\vert\vert\vert$ Δ's	
		$\therefore AH.BG = HC.AB$	
		$\therefore \frac{AH.BG}{HC} = AB$	
		but $\frac{AB}{AC} = \frac{BE}{CF}$ 12.2.2.	
		$\therefore AB = \frac{BE.AC}{CF}$ ✓	
		So, $\frac{AH.BG}{HC} = \frac{BE.AC}{CF}$	6
		——▷	